

Ecuaciones bicuadradas

Matemáticas 3º y 4º de ESO

1. Ecuaciones bicuadradas

1) Calcula las soluciones de esta ecuación bicuadrada

a)

$$x^4 - 5x^2 + 4 = 0$$

$$t = x^2$$

$$t^2 - 5t + 4 = 0$$

$$t = \frac{5 \pm \sqrt{25 - 4(1)(4)}}{2(1)}$$

$$t = \frac{5 \pm \sqrt{25 - 16}}{2}$$

$$t = \frac{5 \pm \sqrt{9}}{2}$$

$$t = \frac{5 \pm 3}{2}$$

$$t_1 = \frac{5 + 3}{2}$$

$$t_1 = \frac{8}{2} \Rightarrow t_1 = 4$$

$$t_2 = \frac{5 - 3}{2} \Rightarrow t_2 = \frac{2}{2} \Rightarrow t_2 = 1$$

$$t = x^2 \Rightarrow x = \sqrt{t}$$

$$t_1 = 4 \Rightarrow x = \sqrt{4} \rightarrow x_1 = +2 \text{ y } x_2 = -2$$

$$t_2 = 1 \Rightarrow x = \sqrt{1} \Rightarrow x_3 = 1 \text{ y } x_4 = -1$$

2) Resuelve esta ecuación bicuadrada

$$x^4 - 5x^2 + 6 = 0$$

$$x^2 = t$$

$$t^2 - 5t + 6 = 0$$

$$t = \frac{5 \pm \sqrt{25 - 4(1)(+6)}}{2(1)}$$

$$t = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$t = \frac{5 \pm 1}{2}$$

$$t_1 = \frac{5+1}{2} \rightarrow t_1 = \frac{6}{2} \Rightarrow t_1 = 3$$

$$t_2 = \frac{5-1}{2} \Rightarrow t_2 = \frac{4}{2} \Rightarrow t_2 = 2$$

$$x^2 = t \Rightarrow x = \sqrt{t}$$

$$t_1 = 3 \Rightarrow x = \sqrt{3} \begin{cases} x_1 = +\sqrt{3} \\ x_2 = -\sqrt{3} \end{cases}$$

$$t_2 = 2 \Rightarrow x = \sqrt{2} \begin{cases} x_3 = +\sqrt{2} \\ x_4 = -\sqrt{2} \end{cases}$$

3) Resuelve esta ecuación incompleta

$$x^4 - 10x^2 + 9 = 0$$

$$x^2 = t$$

$$t^2 - 10t + 9 = 0$$

$$t = \frac{10 \pm \sqrt{100 - 4(1)(9)}}{2(1)}$$

$$t = \frac{10 \pm \sqrt{100 - 36}}{2}$$

$$t = \frac{10 \pm \sqrt{64}}{2}$$

$$t = \frac{10 \pm 8}{2}$$

$$t_1 = \frac{10 + 8}{2} \Rightarrow t_1 = \frac{18}{2} \Rightarrow t_1 = 9$$

$$t_2 = \frac{10 - 8}{2} \Rightarrow t_2 = \frac{2}{2} \Rightarrow t_2 = 1$$

$$t = x^2 \Rightarrow x = \sqrt{t}$$

$$t_1 = 9 \Rightarrow x = \sqrt{9} \begin{cases} x_1 = +3 \\ x_2 = -3 \end{cases}$$

$$t_2 = 1 \Rightarrow x = \sqrt{1} \begin{cases} x_3 = +1 \\ x_4 = -1 \end{cases}$$

4) Resuelve

$$x^4 - 7x^2 = -10$$

$$x^4 - 7x^2 + 10 = 0$$

$$x^2 = t$$

$$t^2 - 7t + 10 = 0$$

$$t = \frac{7 \pm \sqrt{49 - 4(1)(10)}}{2(1)}$$

$$t = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$t = \frac{7 \pm \sqrt{9}}{2}$$

$$t = \frac{7 \pm 3}{2}$$

$$t_1 = \frac{7+3}{2} \Rightarrow t_1 = \frac{10}{2} \Rightarrow t_1 = 5$$

$$t_2 = \frac{7-3}{2} \Rightarrow t_2 = \frac{4}{2} \Rightarrow t_2 = 2$$

$$x^2 = t \Rightarrow x = \sqrt{t}$$

$$t_1 = 5 \Rightarrow x = \sqrt{5} \begin{cases} x_1 = +\sqrt{5} \\ x_2 = -\sqrt{5} \end{cases}$$

$$t_2 = 2 \Rightarrow x = \sqrt{2} \begin{cases} x_3 = +\sqrt{2} \\ x_4 = -\sqrt{2} \end{cases}$$

5) Resuelve:

$$4x^4 - 37x^2 + 9 = 0$$

$$x^2 = t$$

$$4t^2 - 37t + 9 = 0$$

$$t = \frac{37 \pm \sqrt{1369 - 4(4)(9)}}{2(4)}$$

$$t = \frac{37 \pm \sqrt{1369 - 144}}{8}$$

$$t = \frac{37 \pm \sqrt{1225}}{8}$$

$$t = \frac{37 \pm 35}{8}$$

$$t_1 = \frac{37 + 35}{8} \Rightarrow t_1 = \frac{72}{8} \Rightarrow t_1 = 9$$

$$t_2 = \frac{37 - 35}{8} \Rightarrow t_2 = \frac{2}{8} \Rightarrow t_2 = \frac{1}{4}$$

$$x^2 = t \Rightarrow x = \sqrt{t}$$

$$t_1 = 9 \Rightarrow x = \sqrt{9} \Rightarrow \begin{cases} x_1 = +3 \\ x_2 = -3 \end{cases}$$

$$t_2 = \frac{1}{4} \Rightarrow x = \sqrt{\frac{1}{4}} \Rightarrow \begin{cases} x_3 = +\frac{1}{2} \\ x_4 = -\frac{1}{2} \end{cases}$$

6) Resuelve

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$$9x^4 - 37x^2 + 4 = 0$$

$$x^2 = t$$

$$9t^2 - 37t + 4 = 0 -$$

$$t = \frac{37 \pm \sqrt{1369 - 4(9)(4)}}{2(9)}$$

$$t = \frac{37 \pm \sqrt{1369 - 144}}{18}$$

$$t = \frac{37 \pm \sqrt{1225}}{18}$$

$$t = \frac{37 \pm 35}{18}$$

$$t_1 = \frac{37 + 35}{18} \Rightarrow t_1 = \frac{72}{18} \Rightarrow t_1 = 4$$

$$t_2 = \frac{37 - 35}{18} \Rightarrow t_2 = \frac{2}{18} \Rightarrow t_2 = \frac{1}{9}$$

$$x^2 = t \Rightarrow x = \sqrt{t}$$

$$t_1 = 4 \Rightarrow x = \sqrt{4} \Rightarrow \begin{cases} x_1 = +2 \\ x_2 = -2 \end{cases}$$

$$t_2 = \frac{1}{9} \Rightarrow x = \sqrt{\frac{1}{9}} \Rightarrow \begin{cases} x_3 = +\frac{1}{3} \\ x_4 = -\frac{1}{3} \end{cases}$$

7) Resuelve

$$x^4 - 11x^2 + 18 = 0$$

$$t = x^2$$

$$t^2 - 11t + 18 = 0$$

$$t = \frac{11 \pm \sqrt{121 - 4(1)(18)}}{2(1)}$$

$$t = \frac{11 \pm \sqrt{121 - 72}}{2}$$

$$t = \frac{11 \pm \sqrt{49}}{2}$$

$$t = \frac{11 \pm 7}{2}$$

$$t_1 = \frac{11 + 7}{2} \Rightarrow t_1 = \frac{18}{2} \Rightarrow t_1 = 9$$

$$t_2 = \frac{11 - 7}{2} \Rightarrow t_2 = \frac{4}{2} \Rightarrow t_2 = 2$$

$$t = x^2 \Rightarrow x = \sqrt{t}$$

$$t_1 = 9 \Rightarrow x = \sqrt{9} \Rightarrow \begin{cases} x_1 = +3 \\ x_2 = -3 \end{cases}$$

$$t_2 = 2 \Rightarrow x = \sqrt{2} \Rightarrow \begin{cases} x_3 = +\sqrt{2} \\ x_4 = -\sqrt{2} \end{cases}$$

8) Resuelve esta ecuación bicuadrada

$$x^4 = 13x^2 - 36$$

$$x^4 - 13x^2 + 36 = 0$$

$$x^2 = t$$

$$t^2 - 13t + 36 = 0$$

$$t = \frac{13 \pm \sqrt{169 - 4(1)(36)}}{2(1)}$$

$$t = \frac{13 \pm \sqrt{169 - 144}}{2}$$

$$t = \frac{13 \pm \sqrt{25}}{2}$$

$$t = \frac{13 \pm 5}{2}$$

$$t_1 = \frac{13 + 5}{2} \Rightarrow t_1 = \frac{18}{2} \Rightarrow t_1 = 9$$

$$t_2 = \frac{13 - 5}{2} \Rightarrow t_2 = \frac{8}{2} \Rightarrow t_2 = 4$$

$$x^2 = t \Rightarrow x = \sqrt{t}$$

$$t_1 = 9 \Rightarrow x = \sqrt{9} \Rightarrow \begin{cases} x_1 = +3 \\ x_2 = -3 \end{cases}$$

$$t_2 = 4 \Rightarrow x = \sqrt{4} \Rightarrow \begin{cases} x_3 = +2 \\ x_4 = -2 \end{cases}$$

2. Ecuaciones de la familia de las bicuadradas

9) Resuelve esta ecuación de la familia de las bicuadradas

$$8x^6 - 63x^3 - 8 = 0$$

$$t = x^3$$

$$8t^2 - 63t - 8 = 0$$

$$t = \frac{63 \pm \sqrt{3969 - 4(8)(-8)}}{2 \cdot (8)}$$

$$t = \frac{63 \pm \sqrt{3969 + 256}}{16}$$

$$t = \frac{63 \pm \sqrt{4225}}{16}$$

$$t = \frac{63 \pm 65}{16}$$

$$t_1 = \frac{63 + 65}{16} \Rightarrow t_1 = \frac{128}{16} \Rightarrow t_1 = 8$$

$$t_2 = \frac{63 - 65}{16} \Rightarrow t_2 = \frac{-2}{16} \Rightarrow t_2 = -\frac{1}{8}$$

$$t = x^3 \Rightarrow x = \sqrt[3]{t}$$

$$t_1 = 8 \Rightarrow x = \sqrt[3]{8} \Rightarrow x_1 = 2$$

$$t_2 = -\frac{1}{8} \Rightarrow x = \sqrt[3]{-\frac{1}{8}} \Rightarrow x_2 = -\frac{1}{2}$$

10) Resuelve esta ecuación de la familia de las bicuadradas

$$x^6 - 9x^3 + 8 = 0$$

$$t = x^3$$

$$t^2 - 9t + 8 = 0$$

$$t = \frac{9 \pm \sqrt{81 - 4(1)(8)}}{2(1)}$$

$$t = \frac{9 \pm \sqrt{81 - 32}}{2}$$

$$t = \frac{9 \pm \sqrt{49}}{2}$$

$$t = \frac{9 \pm 7}{2}$$

$$t_1 = \frac{9+7}{2} \Rightarrow t_1 = \frac{16}{2} \Rightarrow t_1 = 8$$

$$t_2 = \frac{9-7}{2} \Rightarrow t_2 = \frac{2}{2} \Rightarrow t_2 = 1$$

$$t = x^3 \Rightarrow x = \sqrt[3]{t}$$

$$t_1 = 8 \Rightarrow x_1 = \sqrt[3]{8} \Rightarrow x_1 = 2$$

$$t_2 = 1 \Rightarrow x_2 = \sqrt[3]{1} \Rightarrow x_2 = 1$$

11) Resuelve

$$x^6 - 35x^3 + 216 = 0$$

$$x^3 = t$$

$$t^2 - 35t + 216 = 0$$

$$t = \frac{35 \pm \sqrt{1225 - 4(1)(216)}}{2(1)}$$

$$t = \frac{35 \pm \sqrt{1225 - 864}}{2}$$

$$t = \frac{35 \pm \sqrt{361}}{2}$$

$$t = \frac{35 \pm 19}{2} \left\{ \begin{array}{l} t_1 = \frac{35 + 19}{2} \Rightarrow t_1 = \frac{54}{2} \Rightarrow t_1 = 27 \\ t_2 = \frac{35 - 19}{2} \Rightarrow t_2 = \frac{16}{2} \Rightarrow t_2 = 8 \end{array} \right.$$

$$t = x^3 \Rightarrow x = \sqrt[3]{t}$$

$$t_1 = 27 \Rightarrow x_1 = \sqrt[3]{27} \Rightarrow x_1 = 3$$

$$t_2 = 8 \Rightarrow x_2 = \sqrt[3]{8} \Rightarrow x_2 = 2$$

12) Resuelve esta ecuación

$$x^8 - 97x^4 + 1296 = 0$$

$$t = x^4$$

$$t^2 - 97t + 1296 = 0$$

$$t = \frac{97 \pm \sqrt{9409 - 4(1)(1296)}}{2 \cdot (1)}$$

$$t = \frac{97 \pm \sqrt{9409 - 5184}}{2}$$

$$t = \frac{97 \pm \sqrt{4225}}{2}$$

$$t = \frac{97 \pm 65}{2} \left\{ \begin{array}{l} t_1 = \frac{97 + 65}{2} =]t_1 = \frac{162}{2} \Rightarrow t_1 = 81 \\ t_2 = \frac{97 - 65}{2} \Rightarrow t_2 = \frac{32}{2} \Rightarrow t_2 = 16 \end{array} \right.$$

$$t = x^4 \Rightarrow x = \sqrt[4]{t}$$

$$t_1 = 81 \Rightarrow x = \sqrt[4]{81} \left\{ \begin{array}{l} x_1 = +3 \\ x_2 = -3 \end{array} \right.$$

$$t_2 = 16 \Rightarrow x = \sqrt[4]{16} \left\{ \begin{array}{l} x_3 = +2 \\ x_4 = -2 \end{array} \right.$$

13) Resuelve

$$x^{10} + 31x^5 - 32 = 0$$

$$x^5 = t$$

$$t^2 + 31t - 32 = 0$$

$$t = \frac{-31 \pm \sqrt{961 - 4(1)(-32)}}{2 \cdot (1)}$$

$$t = \frac{-31 \pm \sqrt{1089}}{2}$$

$$t = \frac{-31 \pm 33}{2} \left\{ \begin{array}{l} t_1 = \frac{-31 + 33}{2} \Rightarrow t_1 = \frac{2}{2} \Rightarrow t_1 = 1 \\ t_2 = \frac{-31 - 33}{2} \Rightarrow t_2 = \frac{-64}{2} \Rightarrow t_2 = -32 \end{array} \right.$$

$$t = x^5 \Rightarrow x = \sqrt[5]{t} \left\{ \begin{array}{l} t_1 = 1 \Rightarrow x_1 = \sqrt[5]{1} \Rightarrow x_1 = 1 \\ t_2 = -32 \Rightarrow x_2 = \sqrt[5]{-32} \Rightarrow x_2 = -2 \end{array} \right.$$

14)

15)

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